

Operations with Fractions

In Unit 5 students deepen their understanding of fractions and develop strategies for adding, subtracting, multiplying, and dividing fractions. The first four lessons focus on adding and subtracting fractions and mixed numbers. In these lessons students realize that fractions and mixed numbers are considerably easier to add and subtract when they have a common denominator. They learn various strategies to find and rewrite fractions with common denominators. They use estimation strategies to assess the reasonableness of their answers, and they reflect on which types of problems can be solved mentally and which cannot. Students look for patterns in their solutions to help them develop rules and shortcuts for solving certain types of problems.

Lessons 5-5 through 5-9 focus on fraction multiplication. Students begin by reviewing the “fraction-of” work they did in previous units. They connect this concept to multiplying a fraction and a whole number. Students then use paper folding and area models to multiply fractions by other fractions. They learn a fraction multiplication algorithm that states that the product of two fractions can be found by multiplying the numerators and multiplying the denominators. For example, $\frac{2}{3} * \frac{4}{5} = \frac{(2 * 4)}{(3 * 5)} = \frac{8}{15}$. In Lesson 5-10 students apply their conceptual understanding of fraction multiplication to solve a real-world problem.

In Lessons 5-11 and 5-12 students further apply their understanding of fraction multiplication. In Grade 4 students learned the multiplication rule for equivalent fractions, which states that multiplying the numerator and denominator by the same number gives a fraction that is equivalent to the original fraction. In this unit they explain why the multiplication rule for equivalent fractions works. They also explain why multiplying one number by a number greater than 1 produces a product greater than the original number, while multiplying a number by a fraction less than 1 produces a product less than the original number. Students then explore fraction multiplication in real-world contexts and write their own fraction multiplication number stories. For example, they might write a story about finding a fraction of something or finding the area of a rectangular object that has fractional side lengths.

Lessons 5-13 and 5-14 introduce fraction division. Students use drawings and other visual models to divide unit fractions (fractions with a 1 in the numerator) by whole numbers, solving problems such as $\frac{1}{2} \div 3 = \frac{1}{6}$. Students also divide whole numbers by unit fractions, solving problems such as $3 \div \frac{1}{2} = 6$. They think about the relationship between multiplication and division to check their work. For example, after solving $3 \div \frac{1}{2} = 6$, students multiply $\frac{1}{2} * 6 = 3$ to check that their division is correct.

Vocabulary

Important terms in Unit 5:

algorithm A set of step-by-step instructions for doing something, such as carrying out a computation or solving a problem.

area model A model for multiplication in which the length and width of a rectangle represent the *factors*, and the area of the rectangle represents the *product*.

common denominator Any nonzero number that is a *multiple* of the denominators of two or more fractions. For example, the fractions $\frac{1}{2}$ and $\frac{2}{3}$ have common denominators 6, 12, 18, and other *multiples* of 6. See also **quick common denominator**.

dimension A measure along one direction of an object, typically length, width, or height. For example, the dimensions of a rectangle might be 2 units by 4 units.

dividend In division, the number that is being divided. For example, in $\frac{1}{2} \div 3 = \frac{1}{6}$, the dividend is $\frac{1}{2}$.

divisor In division, the number that divides another number. For example, in $\frac{1}{2} \div 3 = \frac{1}{6}$, the divisor is 3.

equivalent fractions Fractions that name the same number, such as $\frac{1}{2}$, $\frac{4}{8}$, and $\frac{8}{16}$.

factor of a counting number n A counting number whose *product* with some other counting number equals n . For example, 2 and 3 are factors of 6 because $2 * 3 = 6$.

multiple of a counting number n A *product* of n and a counting number. For example, the multiples of 7 are 7, 14, 21, 28, and so on.

multiplication rule for equivalent fractions A rule stating that if the numerator and denominator of a fraction are multiplied by the same non-zero number, the result is a fraction that is equivalent to the original fraction.

product The result of multiplying two numbers, called *factors*. For example, in $4 * 3 = 12$, the product is 12.

quick common denominator (QCD) The *product* of the denominators of two or more fractions. For example, the quick common denominator for $\frac{1}{4}$ and $\frac{3}{6}$ is $4 * 6$, or 24.

quotient The result of dividing one number by another number. For example, in $\frac{1}{2} \div 3 = \frac{1}{6}$, the quotient is $\frac{1}{6}$.

unit fraction A fraction whose numerator is 1, such as $\frac{1}{2}$ or $\frac{1}{7}$.

Do-Anytime Activities

To work with your child on the key concepts in this unit, try some of these activities.

1. Have your child solve simple real-world fraction addition and subtraction problems. For example:
This recipe calls for $\frac{3}{4}$ cup of white flour and $\frac{1}{2}$ cup of wheat flour. How much flour do we need in all? How much more white flour do we need than wheat flour?
2. Point out a collection of objects, such as the 20 slices of bread in a loaf. Ask your child to figure out how many slices would be in a fraction of the loaf. For example, what is $\frac{1}{5}$ of 20 slices or $\frac{2}{3}$ of 20 slices?
3. Have your child fold a sheet of paper to find a fraction of a fraction, such as $\frac{1}{2}$ of $\frac{2}{3}$.

Building Skills through Games

In Unit 5 your child will practice strategies for fraction addition, subtraction, multiplication, and division. He or she will also practice concepts and skills from previous units. Detailed instructions for each game are in the *Student Reference Book*. Many of these games can be played at home with materials you likely already have; gameboards can be copied for home use.

Decimal Top-It: Addition or Subtraction See *Student Reference Book*, pages 298 and 299. Two players need number cards 0–9 (4 of each), 4 counters, and a calculator for this game. *Decimal Top-It: Addition or Subtraction* provides practice adding or subtracting decimals.

Fraction Of See *Student Reference Book*, pages 306–307. Two players need *Fraction Of* cards and a *Fraction Of* record sheet from *Math Masters*, page G24 for this game. *Fraction Of* provides practice with multiplying fractions by whole numbers.

Fraction Top-It: Addition See *Student Reference Book*, page 309. Two players need fraction cards for this game. *Fraction Top-It: Addition* provides practice adding fractions.

Fraction/Whole Number Top-It See *Student Reference Book*, page 310. Two to four players need number cards 1–10 (4 of each) and fraction cards for this game. *Fraction/Whole Number Top-It* provides practice multiplying fractions by whole numbers.

Multiplication or Division Top-It: Larger Numbers See *Student Reference Book*, page 325. Two players need number cards 0–9 (4 of each) for these games. *Multiplication* and *Division Top-It: Larger Numbers* provide practice multiplying and dividing multidigit whole numbers.

As You Help Your Child with Homework

As your child brings assignments home, you might want to go over the instructions together, clarifying them as necessary. The answers listed below will guide you through this unit's Home Links.

Home Link 5-1

1. $\frac{3}{6}, \frac{5}{10}, \frac{4}{6}, \frac{8}{12}, \frac{12}{18}, \frac{9}{12}, \frac{15}{20}$

2. $8; \frac{6}{8} - \frac{4}{8} = ?; \frac{2}{8}$

4. 64,347

3. $6; \frac{1}{6} + \frac{4}{6} = ?; \frac{5}{6}$

5. 65,178

3. $\frac{5}{20} + \frac{4}{20} = \frac{9}{20}$

5. 108 R70

4. $\frac{5}{9} + \frac{6}{9} = \frac{11}{9}$, or $1\frac{2}{9}$

6. 17 R9

Home Link 5-2

1. a. $35; \frac{20}{35}, \frac{21}{35}; <$

c. $20; \frac{5}{20}, \frac{4}{20}; >$

e. $24; \frac{10}{24}, \frac{9}{24}; >$

b. $9; \frac{5}{9}, \frac{6}{9}; <$

d. $18; \frac{14}{18}, \frac{15}{18}; <$

2. $\frac{21}{35} - \frac{20}{35} = \frac{1}{35}$

Home Link 5-3

1. $5\frac{5}{6}$

4. $27\frac{9}{10}$

7. One and forty-six hundredths

8. Four and three hundred nine thousandths

2. $8\frac{3}{6}$

5. 3.624

3. $1\frac{4}{12}$

6. 14.012

Home Link 5-4

- $8\frac{1}{8} - 4\frac{3}{8} = m; 3\frac{6}{8}$
- $12\frac{3}{4} + 5\frac{1}{3} = m; 18\frac{1}{12}$
- $9\frac{1}{8} - 4\frac{1}{2} = m; 4\frac{5}{8}$
- 128.174
- 1,737.405
- 8.25

Home Link 5-5

- 4 gallons; $\frac{2}{3} * 6 = 4$
- 12 books; $\frac{4}{5} * 15 = 12$
- 70 miles; $\frac{7}{10} * 100 = 70$
- 18 square feet; $\frac{3}{4} * 24 = 18$
- < 6. < 7. = 8. > 9. > 10. >

Home Link 5-6

- a. 3 b. 12 c. 12
- a. 45 b. $5\frac{5}{8}$ c. $5\frac{5}{8}$ d. $5\frac{5}{8}$
- $7 * \frac{2}{5} = ?; 2\frac{4}{5}$ 4. $24 * \frac{3}{4} = ?; 18$
- 0.9 6. 0.4 7. 7.1 8. 1.48 9. 5.72 10. 3.00

Home Link 5-7

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- d. $\frac{2}{12}$ e. $\frac{2}{12}$ d. $\frac{6}{12}$ e. $\frac{6}{12}$
- 4.79 4. 9.67 5. 10.04

Home Link 5-8

- $\frac{3}{4} \cdot \frac{1}{2} \cdot \frac{3}{8} \cdot \frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$ 2. $\frac{1}{3} \cdot \frac{2}{3} \cdot \frac{2}{9} \cdot \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$
- 4.31 4. 2.95 5. 2.33

Home Link 5-9

- $\frac{1}{6}$
- $\frac{4}{12}$
- $\frac{8}{25}$
- $\frac{4}{30}$
- $\frac{10}{48}$
- $\frac{10}{84}$
- Less than $\frac{2}{3}$
- Less than $\frac{6}{10}$
- True
- False
- False
- True
- $\frac{5}{6}$
- $\frac{9}{8}$, or $1\frac{1}{8}$
- $\frac{13}{20}$

Home Link 5-10

- Sample answers: $\frac{1}{3} * \frac{2}{4} = \frac{2}{12}; \frac{2}{4} * \frac{1}{3} = \frac{2}{12}$
- Sample answers: $\frac{3}{4} * \frac{2}{5} = \frac{6}{20}; \frac{2}{5} * \frac{3}{4} = \frac{6}{20}$
- $3\frac{6}{8}$ 4. $2\frac{2}{3}$ 5. $7\frac{1}{9}$ 6. $4\frac{7}{12}$

Home Link 5-11

- a. $\frac{4}{4}, \frac{16}{20}$ b. $\frac{6}{6}, \frac{6}{18}$
- a. No. 4. 6 5. 12

Home Link 5-12

- $\frac{8}{3}$, or $2\frac{2}{3}$; Sample answer: Our class ordered 4 kinds of pizza for a celebration. We ate $\frac{2}{3}$ of each pizza. How much pizza did we eat in all?
- 8; Sample answer: There are 16 students playing kickball. Half of them are girls. How many girls are playing kickball?
- 6.82 4. 4.34

Home Link 5-13

- $\frac{1}{2} \div 4 = b; \frac{1}{8}; \frac{1}{2} \div 4 = \frac{1}{8}; \frac{1}{8} * 4 = \frac{4}{8}$, or $\frac{1}{2}$
- $\frac{1}{4} \div 3 = r; \frac{1}{12}; \frac{1}{4} \div 3 = \frac{1}{12}; \frac{1}{12} * 3 = \frac{3}{12}$, or $\frac{1}{4}$
- 22,113 4. 71,568

Home Link 5-14

- $2 \div \frac{1}{8} = b; 16; 2 \div \frac{1}{8} = 16; 16 * \frac{1}{8} = \frac{16}{8} = 2$
- $5 \div \frac{1}{2} = m; 10; 5 \div \frac{1}{2} = 10; 10 * \frac{1}{2} = \frac{10}{2} = 5$
- 77 R7 4. 89 R45